Anomalous transport in unbound and ratchet potentials

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A non-Markovian Langevin equation with a broadband noise is proposed to describe anomalous transport of a particle passing over a potential saddle or moving in a ratchet potential. In the presence of thermal broadband noise, the asymptotic mean square displacement of a free particle is proportional to the square of time; this is called ballistic diffusion. The passing probability of a particle driven by this broadband noise over the saddle of an inverted harmonic potential is obtained analytically. It is shown that the passing probability increases with the kinetic energy, which is slower than that of normal case. The mechanisms of ballistic diffusion and mobility are also applied to the rocking (a square-wave driving force acting on the potential) and flashing (the potential fluctuating between on and off) ratchets. Phenomena such as acceleration and double-peak mean velocity are observed.

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I. INTRODUCTION

Recently, there has been a great deal of renewed interest in the problems of how motion of a particle is affected by the dissipative influence of a disordered heat bath. The disorder of the background medium may induce a memory effect of velocity into the diffusion process [1–8]. The particle experiences anomalous diffusion in the force-free case, its mean-squared displacement in long times reads $\langle q^2(t)\rangle \propto t^\delta$, $0<\delta<1$ for subdiffusion; $\delta=1$ for normal diffusion; $1<\delta<2$ for superdiffusion; and $\delta=2$ is called ballistic diffusion [6]. Superdiffusion has been found in a number of systems [9] ranging from early discoveries in intermittent chaotic systems, fluid particles in fully developed turbulence, and millennial climate changes. So far, there remain open questions, such as the novel features of anomalous diffusion in unbound potentials, suppression of barrier diffusion, and diffusion helping drift, etc.

In classical mechanics, a particle can climb up to the top of a potential if its initial kinetic energy is equal to the energy difference between the potential top and its initial position. Thus the probability of the particle passing over the saddle is a step function of the initial kinetic energy. However, for dissipative systems, if the probability of the particle passing over the potential saddle is larger than $\frac{1}{2}$, the initial kinetic energy of the particle must be much larger than the energy difference between the potential top and the initial position. This is because a part of the energy dissipates into internal degrees of freedom [10–12]. For instance, for the fusion process of a massive heavy-ion system, as a typical directional diffusion, extra-push energy is necessary in addition to the Coulomb barrier. Further, in comparison with the experimental data, the fusion probability curves with increasing center-of-mass energy are steep in the calculations of both the WKB and the preliminary diffusive model. Normal diffusion cannot be applied to analyze these phenomena well; the anomalous diffusive mechanism needs to be added.

The anomalous diffusion and Lévy flights in periodic potentials have attracted little attention in the past [13]. For a particle moving in an asymmetrical periodic potential, the dynamical effect of anomalous diffusion and mobility should be very interesting. An excellent probe for directional periodic motion may be the Brownian ratchet related to molecular motors [14]. Motor proteins move cell organelles along the cytoskeleton; their motions are random, but directed on average. Adenosine triphosphate (ATP) coupling to a motor protein induces a series of conformational changes. These are modeled by transitions between different states; alternatively one can consider a protein moving in a fluctuation potential [15]. The presence of ATP might lead to a nonhomogeneous background, or a disordered media, thus anomalous diffusion exists when the potential is absent. If the potential is recovered, the right and left probabilities across the positions of two near barriers are different, and this gives rise to a drift [16,17]. It is obvious that diffusion influences directional motion.

The purpose of this paper is to study how ballistic diffusion, as a strong superdiffusion, influences directional transport, and a comparison with normal diffusion is also performed. It is believed that the present work can help one to learn characteristic behaviors of both barrier passage problem and nonequilibrium fluctuation-induced directed motion. The paper is organized as follows. In Sec. II, a thermal broadband noise is discussed, which can induce ballistic diffusion, and a non-Markovian Langevin equation (NMLE) is transformed into a set of Markovian Langevin equations. In Sec. III, the problem of directional diffusion passing over the saddle of an inverse harmonic potential is added, and the exact expression for the passing probability is obtained. In Sec. IV, directed motions of a particle in the rocking and flashing ratchets in the presence of ballistic diffusion and mobility are studied. A summary is given in Sec. V.

II. NMLE WITH A THERMAL BROADBAND NOISE

The NMLE of the motion of a particle reads

$$m\ddot{q}(t)+m\int_0^t \beta(t-t')\dot{q}(t')dt'+U'(q)=\varepsilon(t)+E(t), \tag{1}$$

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where $q(t)$ is the coordinate of the particle, $\beta(t)$ is the friction memory kernel, $\varepsilon(t)$ is a zero-mean, stationary thermal noise and obeys the fluctuation-dissipation theorem: $\langle\varepsilon(t)\varepsilon(t')\rangle = k_B T\beta(|t-t'|)$, k_B is the Boltzmann constant and T the absolute temperature of the environment, and $E(t)$ is an external unbiased fluctuation.

The long-time diffusive behavior of the particle is controlled by the low-frequency limit of the noise density of states [6]. In order to generate a deep superdiffusion, such as ballistic diffusion, we need to remove the lower-frequency part of the spectral density $S(\omega)$ of the noise, i.e., $S(0) = 0$. Here the spectral density is defined by the Fourier transformation of the correlation function of the noise: $S(\omega) = \int_0^\infty \exp(-i\omega\tau)\langle\varepsilon(\tau)\varepsilon(0)\rangle d\tau$. It is clear that the spectral density of the harmonic noise has a local minimum at $\omega=0$ [18], but it is not vanishing in the low-frequency limit. Thus this noise cannot induce a superdiffusion. We know that the spectral density of the Ornstein-Uhlenbeck noise (OUN) equals a constant at zero frequency and decays at high frequency. If the difference between two OUNs with different time constants, driven by the same white noise, is regarded as a noise source, its zero frequency vanishes and it exhibits a peak at a finite frequency. Thus a broadband spectrum is realized by this noise [19]. We propose

$$\varepsilon(t) = \varepsilon_2(t) - \varepsilon_1(t),$$

$$\dot{\varepsilon}_j(t) = -\frac{1}{\tau_j}\varepsilon_j(t) + \frac{1}{\tau_j}\xi(t) \quad (j=1,2),$$

$$\langle\xi(t)\rangle = 0, \quad \langle\xi(t)\xi(t')\rangle = 2D\delta(t-t'), \tag{2}$$

where τ_1 and τ_2 are correlation times of the noise, and D is the intensity of white noise $\xi(t)$.

If we assume that $\langle\varepsilon_j^2(0)\rangle = D/\tau_j$ $(j=1,2)$ and $\langle\varepsilon_1(0)\varepsilon_2(0)\rangle = 2D/(\tau_1+\tau_2)$, the noise $\epsilon(t)$ is a stationary process at any time and its correlation function reads

$$\langle\varepsilon(t)\varepsilon(t')\rangle = \frac{D(\tau_1-\tau_2)}{\tau_1+\tau_2}\left\{\frac{1}{\tau_2}\exp\left(-\frac{|t-t'|}{\tau_2}\right) - \frac{1}{\tau_1}\exp\left(-\frac{|t-t'|}{\tau_1}\right)\right\}. \tag{3}$$

In order to allow a transition between low-passing "red" noise and high-passing "green" noise [20], we choose $D = \beta_0 k_B T[\tau_1/(\tau_1-\tau_2)]^2$, where β_0 is the friction coefficient. We introduce a dimensionless quantity from the spectral density divided by $\beta_0 k_B T$: $C(\omega) = S(\omega)/(\beta_0 k_B T)$. For the present broadband noise, we have

$$C_b^{-1}(\omega) = (\tau_1\omega)^{-2} + \left(1+\frac{\tau_2^2}{\tau_1^2}\right) + (\tau_2\omega)^2. \tag{4}$$

$C_b(\omega)$ has a peak at the frequency $\omega_p = (\tau_1\tau_2)^{-1/2}$. The spectrum of this noise displays a broadband and has rich high frequencies in comparison with the harmonic noise. When $\tau_1\to\infty$, $C_b(\omega) = (1+\tau_2^2\omega^2)^{-1}$ shows a red spectrum; when $\tau_2\to 0$, $C_b(\omega) = [1+(\tau_1\omega)^{-2}]^{-1}$ shows a green spectrum, because $C_b(\omega)$ is equal to a homogeneous spectrum of the white noise plus a low-passing spectrum of the red noise.

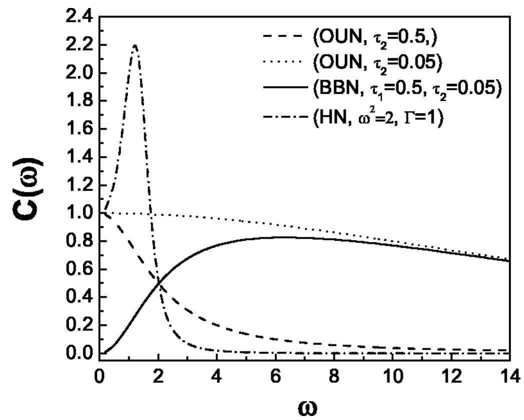

FIG. 1. The spectral densities divided by $(\beta_0 k_B T)$ of three kinds of colored noises: Ornstein-Uhlenbeck noise (OUN); harmonic noise (HN); and broadband noise (BBN).

The harmonic noise is named quasimonochromatic noise; the inverse of its spectrum divided by $(\beta_0 k_B T)$ is given by

$$C_h^{-1}(\omega) = \omega_0^{-4}[(\omega^2-\omega_0^2)^2 + \Gamma^2\omega^2]. \tag{5}$$

$C_h(\omega)$ has a sharp peak at the frequency $(\omega_0^2 - \frac{1}{2}\Gamma^2)^{1/2}$. The harmonic noise can be viewed as the result of passing white noise $\xi(t)$ through a harmonic oscillator filter: $\ddot{\zeta} + \Gamma\dot{\zeta} + \omega_0^2 = \xi(t)$ [18]. The spectral densities of the above three kinds of colored noises are shown in Fig. 1.

If one directly simulates the NMLE (1) with the noise process (2), double numerical integrations for the memory velocity are required, thus the run time is not only long but iterative errors also collect and develop in the final results. Now we transform Eq. (1) into a set of Markovian Langevin equations (MLE) by introducing two variables

$$y_j(t) = (-1)^j\left[-\frac{A}{\tau_j}\int_0^t \exp\left(-\frac{t-s}{\tau_j}\right)\dot{q}(s)ds + \varepsilon_j(t)\right](j=1,2), \tag{6}$$

where $A = \beta_0\tau_1^2/(\tau_1^2-\tau_2^2)$. Thus a set of MLE is yielded as

$$m\ddot{q}(t) + U'(q) = \sum_{j=1}^{2} y_j(t) + E(t),$$

$$\dot{y}_j(t) + \frac{1}{\tau_j}y_j(t) = \frac{(-1)^j}{\tau_j}[-A\dot{q}(t) + \xi(t)]. \tag{7}$$

Here, the initial distributions of the two stochastic variables y_1 and y_2 are the same as that of ε_1 and ε_2, respectively.

III. BARRIER PASSAGE PROBLEM

We apply Eq. (1) to the saddle passing problem [10–12,21]. Assuming that the initial position of the particle is close to the saddle point of a potential, then around this point, the potential is expressed to be an inverse harmonic potential:

$$U(q) = -\tfrac{1}{2} m\Omega^2 q^2, \tag{8}$$

where the coordinate origin is taken at the potential top. In this case, the exact solution of Eq. (1) can be obtained by using the Laplace transform technique, giving

$$q(t) = \langle q(t)\rangle + \frac{1}{m}\int_0^t H(t-t')\varepsilon(t')dt', \tag{9}$$

where $\langle q(t)\rangle$ is the mean position of the particle given by

$$\langle q(t)\rangle = \left[1+\Omega^2\int_0^t H(t')dt'\right]q_0 + H(t)v_0, \tag{10}$$

where q_0 and v_0 are the initial coordinate and velocity of the particle, respectively.

The Laplace transform of the response function $H(t)$ is

$$\hat{H}(z) = \frac{1}{z^2 + z\hat{\beta}[z] - \Omega^2}, \tag{11}$$

where $\hat{\beta}[z]$ is the Laplace transform of the damping kernel $\beta(t)$. The response function $H(t)$ is the inverse form of the Laplace transform $\hat{H}(s)$. The expression of $\hat{H}(z)$ is given by

$$\hat{H}(z) = \frac{(1+z\tau_1)(1+z\tau_2)}{\Xi}, \tag{12}$$

where

$$\Xi = \tau_1\tau_2 z^4 + (\tau_1+\tau_2)z^3 + (1+\beta'\tau_1 - \Omega^2\tau_1\tau_2)z^2 - \Omega^2(\tau_1+\tau_2)z - \Omega^2 \tag{13}$$

with $\beta' = \beta_0\tau_1/(\tau_1+\tau_2)$.

Applying the residue theorem

$$H(t) = \frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}\hat{H}(s)\exp(st)ds, \tag{14}$$

we obtain the response function

$$H(t) = \sum_{j=1}^{4}\frac{(1+z_j\tau_1)(1+z_j\tau_2)}{(z_j-z_l)(z_j-z_k)(z_j-z_n)}\exp(z_j t), \tag{15}$$

where z_j are the roots of the equation $\Xi=0$; $l,k,n = 1,\ldots,4$ and $l\neq k\neq n\neq j$.

The variance of the coordinate of the particle is calculated by

$$\sigma_q^2(t) = 2k_BT\int_0^t dt_1 H(t-t_1)\int_0^{t_1} dt_2 H(t-t_2)\beta(t_1-t_2). \tag{16}$$

If $\Omega^2=0$, Eq. (13) has double roots of $z=0$, leading to $\sigma_q^2(t)\propto t^2$ in the long-time limit; this phenomenon is called ballistic diffusion.

To evaluate the probability of the particle passing over the potential saddle, one needs a Gaussian distribution Π for the position of the particle at any time [10–12], because here the noise obeys a Gaussian distribution and the potential is a parabolic one. We have

$$\Pi(q,t;q_0,v_0) = \frac{1}{\sqrt{2\pi}\sigma_q(t)}\exp\left(-\frac{[q-\langle q(t)\rangle]^2}{2\sigma_q^2(t)}\right). \tag{17}$$

The ratio of the particles' number N^+ $(q>0)$ having passed the potential saddle to the initial localized particles' number N_0, also called the passing probability, as a function of time is determined by

$$n^+(t;q_0,v_0) = \frac{N^+}{N_0} = \int_0^\infty \Pi(q,t;q_0,v_0)dq = \frac{1}{2}\mathrm{erfc}\left(-\frac{\langle q(t)\rangle}{\sqrt{2}\sigma_q(t)}\right). \tag{18}$$

Defining the critical velocity $v_{0,c}$, necessary to have the passing probability equal to $\frac{1}{2}$, it is obvious that it corresponds to $\lim_{t\to\infty}\langle q(t)\rangle = 0$. From Eq. (10) it can be easily shown that

$$v_{0,c} = -q_0\lim_{t\to\infty}\frac{1+\Omega^2\int_0^t H(t')dt'}{H(t)} = -q_0\frac{\Omega^2}{a}, \tag{19}$$

where a is the largest positive root of Eq. (13). In this case, the critical kinetic energy is

$$K_c = \frac{1}{2}mv_{0,c}^2 = \left(\frac{\Omega}{a}\right)^2 B, \tag{20}$$

and $B=\frac{1}{2}m\Omega^2 q_0^2$ is the difference of potential energy between the saddle point $q=0$ and the initial position $q=q_0$ of the particle.

In the long-time limit,

$$\lim_{t\to\infty}\frac{\langle q(t)\rangle}{\sqrt{2}\sigma_q(t)} = \frac{\frac{\Omega^2}{a}q_0 + v_0}{\left\{2\frac{k_BT}{m}\frac{\beta_0\tau_1^2}{(\tau_1+\tau_2)(1+a\tau_1)(1+a\tau_2)}\right\}^{1/2}}. \tag{21}$$

The stationary passing probability $n_{st}^+ = n^+(t\to\infty;q_0,v_0)$ is then known as a function of the initial kinetic energy and the temperature for a given q_0. It increases from 0 to 1 around the critical value K_c when increasing the initial kinetic energy. The higher the temperature is, the smoother this increase is. For normal case ($\tau_1\to\infty$ and $\tau_2\to 0$) induced by a white noise, $a=[-\beta_0+\sqrt{\beta_0^2+4\Omega^2}]/2$ [11].

The use of a scaling of the form by means of the energy unit k_BT and

$$\tilde{q} = q\Big/\sqrt{\frac{k_BT}{m\Omega^2}},\quad \tilde{t}=\beta_0 t,\quad \tilde{\tau}_j=\beta_0\tau_j,\quad \tilde{\Omega}=\Omega/\beta_0,$$

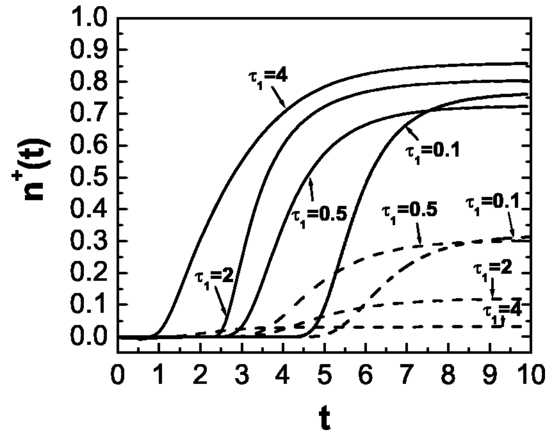

FIG. 2. Time-dependent passing probability $n^+(t)$ for different τ_1 and v_0. The parameters used are $\Omega^2=1.0$, $x_0=-0.5$, $k_BT=1.0$, and $\tau_2=0.05$. The solid and dashed lines take $v_0=1.0$ and $v_0=0.2$, respectively.

$$\tilde{v}_0=v_0\Big/\sqrt{\frac{k_BT}{m}},\quad \tilde{B}=B/(k_BT)$$

leads to a dimensionless formulation in the Eqs. (18) and (21). In numerical calculation the tildes are omitted.

In Fig. 2, we plot the time-dependent passing probability for various τ_1 at a fixed $\tau_2=0.05$. If the initial velocity of the particle is much larger than its critical value, the passing probability could increase from 0 to 1. It is seen that the increase of the passing probability curve becomes rapid with increasing τ_1; this corresponds to the noise changing into whiteness from greenness. Note that the value of a appearing in Eqs. (19)–(21) is a nonmonotonous function of τ_1 at a fixed τ_2, for instance, the noise shows the strongest greenness when $\tau_1=0.5$ in the figure, so that the stationary passing probability is the smallest or the largest for a large or small initial velocity, respectively. This implies that the present thermal broadband noise inhibits directional barrier passage process of the particle with a large initial velocity.

The stationary passing probability n_{st}^+ as a function of the initial kinetic energy $K_0=\frac{1}{2}mv_0^2$ is plotted in Fig. 3. It is seen that the increase of the passing probability by increasing the initial kinetic energy is rather slow for the present noise-induced diffusion, because the thermal green noise has led to a strong randomness. If the temperature increases, thermal diffusion helps the particle with a small v_0 to pass over the potential top. On the other hand, diffusion hinders directional motion of the particle when $v_0>v_{0,c}$.

IV. DIRECTED MOTION IN RATCHETS

Now we consider a Brownian motor in the rocking ratchet [22–24] or the flashing ratchet [15,17,25,26] in the presence of ballistic diffusion. The normal thermal rocking ratchet has been studied in Refs. [27] and [28]. The ratchet potential is chosen to be a piecewise linear potential as

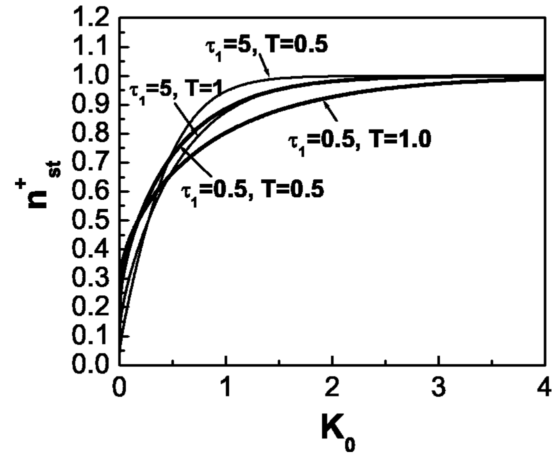

FIG. 3. The stationary passing probability n_{st}^+ as a function of the initial kinetic energy K_0 at a fixed $\tau_2=0.02$ for different τ_1 and k_BT; the other parameters are the same as Fig. 2.

$$U(q)=\begin{cases}\dfrac{U_0}{(1-\alpha)\lambda}q, & 0<q<(1-\alpha)\lambda\\[2mm] \dfrac{U_0}{\alpha\lambda}(\lambda-q), & (1-\alpha)\lambda<q<\lambda,\end{cases}\tag{22}$$

where U_0, α, and λ are barrier height, asymmetrical parameter, and periodic length of the ratchet potential, respectively.

The use of a scaling of the form [29,30],

$$\tilde{q}=q/\lambda,\quad \tilde{t}=t/t_0,\quad \tilde{y}_j=y_jU_0/\lambda\quad (j=1,2),$$

$$U(\tilde{q})=U(q)/U_0,$$

leads to a dimensionless formulation of the dynamics in a potential U with $U(\tilde{q})=U(\tilde{q}+1)$. We choose $t_0=\beta_0\lambda^2/U_0$ to obtain a dimensionless friction coefficient equal to one. Then, the rescaled mass and noise parameters are given as

$$\mu=\frac{mU_0}{\lambda^2\beta_0^2},\quad \tilde{D}=\frac{D}{U_0\beta_0},\quad \tilde{\tau}_j=\frac{\tau_jU_0}{\beta_0\lambda^2},\tag{23}$$

where $j=1,2$. The dimensionless dynamics reads

$$\dot{q}=v,$$

$$\mu\dot{v}=-U'(q)+\sum_{j=1}^{2}y_j(t)+E(t),\tag{24}$$

$$\dot{y}_j=-\frac{y}{\tau_j}+\frac{(-1)^j}{\tau_j}[-Av(t)+\xi(t)],$$

where the tildes are omitted here and later on. We use the stochastic Runge-Kutta algorithm [31] with a small time step $\Delta t=10^{-3}$ to simulate numerically Eq. (24).

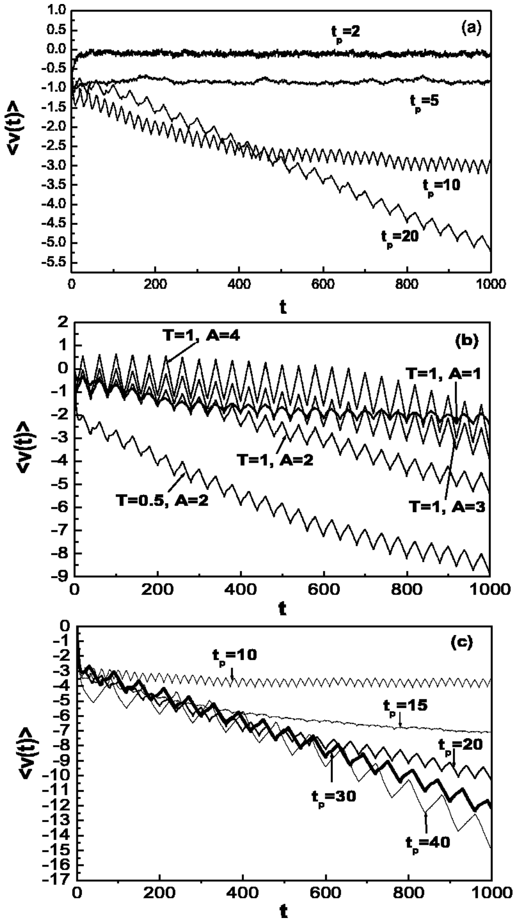

FIG. 4. Time-dependent mean velocity of the particle. The common parameters are $\mu=1.0$, $\tau_1=0.5$, and $\tau_2=0.05$. (a) Different half-cycle periods t_p at fixed $\alpha=0.8$, $k_BT=1.0$, and $A=2.0$; (b) different amplitudes of the driving force at fixed $\alpha=0.8$, $\mu=1.0$, $k_BT=1$, and $t_p=20.0$; (c) different t_p at fixed $\alpha=0.9$, $k_BT=0.5$, and $A=2$.

A. Acceleration in the rocking ratchet

The external driving $E(t)$ is chosen as a square-wave shape of the driving force with amplitude A [24],

$$E(t)=\begin{cases} A, & 2nt_p\leqslant t<(2n+1)t_p \\ -A, & (2n+1)t_p\leqslant t<2(n+1)t_p, \end{cases} \tag{25}$$

where the time period $2t_p$ is assumed to be larger than the time scale of Brownian particles in a bath environment, but smaller than the diffusive time of the particle over the potential barriers.

The quantity of central interest is the time-averaged velocity of the particle in the past, which can be directly evaluated from Eqs. (22), (24), and (25) via numerical simulations. In the normal case, the mean velocity versus A and T has been discussed previously, so here we focus on the characteristic behaviors of directed acceleration due to mobility corresponding to ballistic diffusion. In Figs. 4(a)–4(c), we plot the time-dependent mean velocities of the particle for different half periods t_p of the cycle, amplitudes of the force, temperatures, and asymmetries of the ratchet. The important

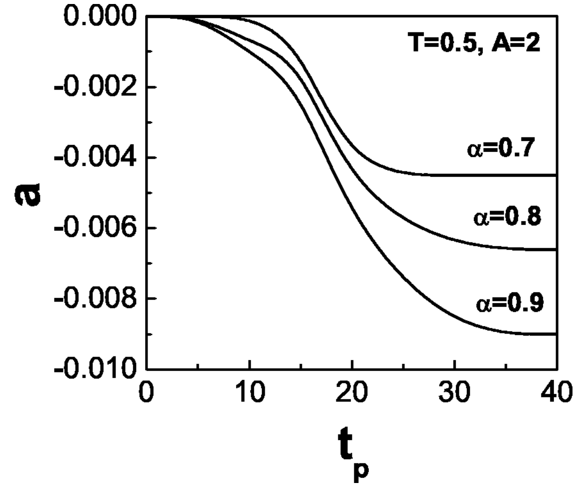

FIG. 5. The acceleration as a function of the half period of the cycle for different asymmetries of the rocking ratchet at fixed $k_BT=0.5$, $\tau_1=0.5$, and $\tau_2=0.05$.

finding is that the mean velocity averaging to a period of the driving force increases linearly with time if $t_p\geqslant 10$, namely, there exists a directed acceleration. Here the mean acceleration of the particle is determined numerically by the slope of the mean velocity,

$$a=\frac{1}{t_f-t_i}[\langle v(t_f)\rangle-\langle v(t_i)\rangle], \tag{26}$$

where t_i and t_f are the beginning and final calculating times of the average velocity increasing linearly with time, as well as $t_f-t_i=2nt_p$, n integral. It is noticed that the zero-frequency friction of the system vanishes [i.e., $\int_0^\infty\beta(t)dt\equiv 0$] and the ratchet potential has an asymmetrical behavior in the present model. The former gives rise to acceleration of the particle along the right and left tilted forces, and the latter leads to the net difference between the directed accelerations in the two directions.

In Fig. 5, the mean acceleration of the particle is shown as a function of the half period t_p of the cycle. For a very small t_p, the mobility of the particle does not have a chance to get started in the $E(t)=-A$ state quickly before a transition back to the pinned $E(t)=A$ state, and so the velocity decreases, where only the directed mean velocity exists. The mean acceleration approaches zero as the frequency becomes infinite, $t_p\to 0$. For large enough A, the total potential $U(q)\pm Aq$ has no local maxima, the mean displacement of the particle is proportional to the square of time and the slope of the tilted potential. For large amplitudes of the driving force, we have $\langle q(t)\rangle\propto F_{eff}t^2$ in the long-time limit, where F_{eff} is the effective tilted force of the two sawtooth sides. Of the four slopes involved in the $E(t)=A$ and $E(t)=-A$ states [24], with $\frac{1}{2}<\alpha<1$, the values of F_{eff} are equal to $U_0/\alpha-A$, $-U_0/(1-\alpha)-A$, $-U_0/(1-\alpha)+A$, and $U_0/\alpha+A$, respectively. The smallest slope is the left shorter side, and this leads to $a(A)<a(-A)$. We thus have a net negative acceleration $\frac{1}{2}[a(A)+a(-A)]$ to the left in the limit of low frequency, and the slope of the mean velocity

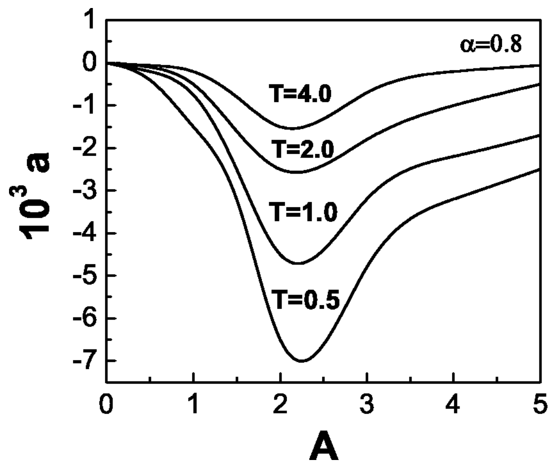

FIG. 6. The acceleration in the adiabatic limit as a function of the amplitude A for different temperatures at fixed $\alpha=0.8$, $\mu=1.0$, $t_p=20.0$, $\tau_1=0.5$, and $\tau_2=0.05$.

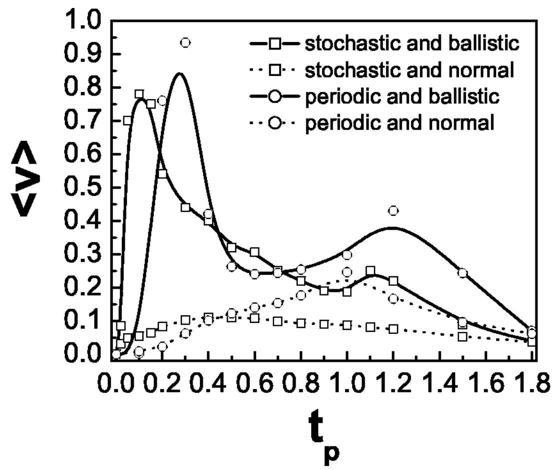

FIG. 7. The comparison of the mean velocity in periodically and stochastically flashing ratchets. The parameters used are $\alpha=0.9$, $\mu=1.0$, $k_BT=0.01$, $\tau_1=0.5$, and $\tau_2=0.05$.

varying with time can arrive at a maximum in the adiabatic approximation for the driving force. The cycle period of the rocking force plays an important role in the appearance of the directed acceleration, and we have found through numerical calculations that the acceleration does not exist if $t_p<10$ even for large A.

Dependence of the acceleration on the amplitude of the rocking force for different temperatures is shown in Fig. 6 in the adiabatic approximation for the driving force. If the amplitude of the driving force is very small, mobility of the particle in the tilted potential is small. As the amplitude of the driving force increases, so does the acceleration. At very large amplitudes of the driving force, however, the effect of ratchet vanishes so that the average acceleration, as the difference between the accelerations along the two directions, decreases, approaching zero as the amplitude of the driving force becomes infinite.

B. Multipeaked flux in the flashing ratchet

Here the external fluctuation in Eq. (1) is taken to be $E(t)=[1-z(t)]U'(q)$, where $z(t)$ is a two-state process taking two values: 0 and 1. If it is a stochastic dichotomous process with transition rate t_p^{-1}, the change of state probability obeys the following random telegraph equation [32]:

$$\partial_t P(0,t|z,0)=t_p^{-1}[-P(0,t|z,0)+P(1,t|z,0)],$$

$$\partial_t P(1,t|z,0)=t_p^{-1}[P(0,t|z,0)-P(1,t|z,0)]. \tag{27}$$

If it is a periodical dichotomous process, and the particle expresses the waiting times t_{off} in the potential off and t_{on} in the potential on, we have

$$z(t)=\begin{cases}0, & 2nt_p<t<2nt_p+t_{off}\\ 1, & 2nt_p+t_{off}<t<2(n+1)t_p,\end{cases} \tag{28}$$

where $t_{off}+t_{on}=2t_p$. Different waiting times of the potential on and off have been considered in Ref. [26], but here we choose $t_{off}=t_{on}=t_p$ only.

For the flashing ratchet, the acceleration does not occur, and the mechanism of fluctuating barrier-induced directional drift differs with that of the rocking ratchet; the former is due to the net difference between diffusive probabilities in the two directions. In Fig. 7, we show the steady mean velocity of the particle in the flashing ratchet as a function of the half period t_p of the cycle and compare it with the result of white noise by the use of the same parameters $\alpha=0.9$ and $k_BT=0.01$. A different phenomenon is that the mean velocity curve has two peaks in the presence of ballistic diffusion. The first peak corresponds to a maximum probability drift during a short period of the cycle and the second peak is due to the particle having enough time to descend to the bottom of a well along the two sawtooth sides, thus it can move a long net distance. It is observed that the maximum of the mean velocity driven by the proposed thermal broadband noise is much larger than that of the white noise.

Moreover, the mean velocity $\langle v\rangle$ in the stochastically flashing ratchet (SFR) is larger than in the periodically flashing ratchet (PFR) for fast cycles; however, $\langle v\rangle_{SFR}<\langle v\rangle_{PFR}$ for slow cycles. This can be understood from the viewpoint of the sudden changing of the probability distribution of the particle. If the waiting time in the potential off is short, the particle does not have sufficient time to cross the nearest barrier; however, for the SFR, the probability of the potential off being switched back the potential on is equal to $\frac{1}{2}$, namely, the particle still has the chance to diffuse. On average, in this case, $\langle v\rangle_{SFR}>\langle v\rangle_{PFR}$. In the opposite situation of sufficiently large t_p, the periodically flashing ratchet is advantageous to the net diffusion of the particle, so that the mean velocity in the PFR is larger than in the SFR for long t_p.

V. CONCLUSIONS

In this work, we study transport of a particle moving in two kinds of unbound potentials such as an inverse harmonic potential and a ratchet potential with a piecewise linear shape. In the presence of a thermal broadband noise, the behavior of the particle passing the saddle of an inverse harmonic potential is studied analytically. The stronger the diffusion is, the more random the motion is, thus the increase of the passing probability with the initial kinetic energy in the present non-Markovian case is slower than that in the normal case; namely, at low kinetic energies, strong diffusion helps the particles to overpass the saddle point, however, if the particle has a large initial kinetic energy, its directional motion is inhibited by strong diffusion.

The case is opposite in the ratchet systems where ballistic diffusion can advance directional motion. In the rocking ratchet, due to the fact that the mean-squared displacement of the particle is proportional to the square of time when both the amplitude and the cycle time period of the driving force are large enough, the particle exhibits a directed acceleration. The acceleration arrives at a maximum in the adiabatic approximation for the driving force, and there is always an optimal temperature and amplitude of the driving force. In the flashing ratchet, the mean velocity is such that as the difference between the right and left diffusive probabilities, the optimal mean velocity results from a cooperation between free diffusion of the particle in the potential off and mobility along the two sawtooth sides in the potential on. The challenge for generalizing this study to an overcoming of a general potential shall be considered in the future.

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